what is needed is a probe that reaches deep into the nucleus, and we must use that probe to measure a nuclear property that characterizes the interior of the nucleus and not its surface. For a probe we choose high-energy electrons, as we did in studying the nuclear charge distribution in Chapter 3. The property that is to be measured is the charge density of a single nucleon in its orbit, which is equivalent to the square of its wave function, $|\psi|^2$. Reviewing Figure 2.12 we recall that only s-state wave functions penetrate deep into the nuclear interior; for other states $\psi \to 0$ as $r \to 0$. For our experiment we therefore choose a nucleus such as $^{205}_{81}\mathrm{Tl}_{124}$, which lacks a single proton in the $3s_{1/2}$ orbit from filling all subshells below the $Z = 82$ gap. How can we measure the contribution of just the $3s_{1/2}$ proton to the charge distribution and ignore the other protons? We can do so by measuring the *difference* in charge distribution between $^{205}\mathrm{Tl}$ and $^{206}_{82}\mathrm{Pb}_{124}$, which has the filled proton shell. Any difference between the charge distributions of these two nuclei must be due to the extra $3s_{1/2}$ proton in $^{206}\mathrm{Pb}$. Figure 5.13 shows the experimentally observed difference in the charge distributions as measured in a recent experiment. The comparison with $|\psi|^2$ for a 3s wave function is very successful (using the same harmonic oscillator wave function plotted in Figure 2.12, except that here we plot $|\psi|^2$, *not* r^2R^2), thus confirming the validity of the assumption about nucleon orbits retaining their character deep in the nuclear interior. From such experiments we gain confidence that the independent-particle description, so vital to the shell model, is not just a convenience for analyzing measurements near the nuclear surface, but instead is a valid representation of the behavior of nucleons throughout the nucleus.

5.2 EVEN-*Z*, EVEN-*N* NUCLEI AND COLLECTIVE STRUCTURE

Now let's try to understand the structure of nuclei with even numbers of protons and neutrons (known as *even-even* nuclei). As an example, consider the case of $^{130}\mathrm{Sn}$, shown in Figure 5.14. The shell model predicts that all even-even nuclei will have 0^+ (spin 0, even parity) ground states, because all of the nucleons are paired. According to the shell model, the 50 protons of $^{130}\mathrm{Sn}$ fill the $g_{9/2}$ shell and the 80 neutrons lack 2 from filling the $h_{11/2}$ shell to complete the magic number of $N = 82$. To form an excited state, we can break one of the pairs and excite a nucleon to a higher level; the coupling between the two odd nucleons then determines the spin and parity of the levels. Promoting one of the $g_{9/2}$ protons or $h_{11/2}$ neutrons to a higher level requires a great deal of energy, because the gap between the major shells must be crossed (see Figure 5.6). We therefore expect that the major components of the wave functions of the lower excited states will consist of neutron excitation within the last occupied major shell. For example, if we assume that the ground-state configuration of $^{130}\mathrm{Sn}$ consists of filled $s_{1/2}$ and $d_{3/2}$ subshells and 10 neutrons (out of a possible 12) occupying the $h_{11/2}$ subshell, then we could form an excited state by breaking the $s_{1/2}$ pair and promoting one of the $s_{1/2}$ neutrons to the $h_{11/2}$ subshell. Thus we would have one neutron in the $s_{1/2}$ subshell and 11 neutrons in the $h_{11/2}$ subshell. The properties of such a system would be determined mainly by the coupling of the $s_{1/2}$ neutron with the unpaired $h_{11/2}$ neutron. Coupling angular momenta j_1 and j_2 in quantum mechanics gives values from the sum $j_1 + j_2$ to

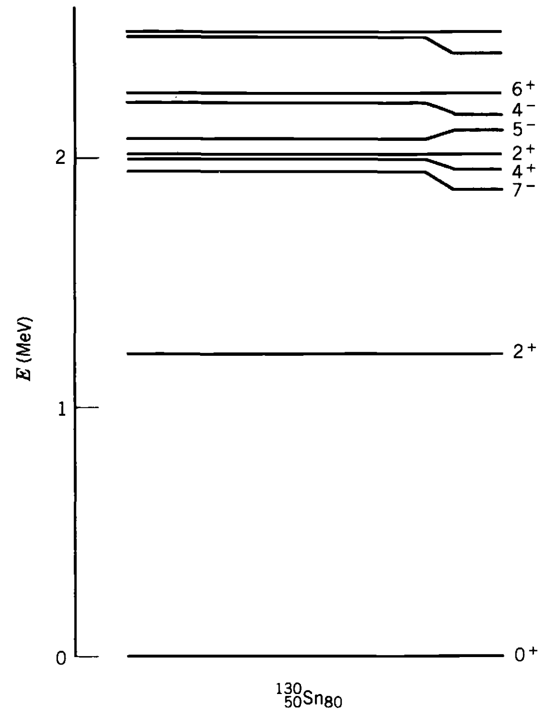

Figure 5.14 The low-lying energy levels of ^{130}Sn.

the difference $|j_1 - j_2|$ in integer steps. In this case the possible resultants are $\frac{11}{2} + \frac{1}{2} = 6$ and $\frac{11}{2} - \frac{1}{2} = 5$. Another possibility would be to break one of the $d_{3/2}$ pairs and again place an odd neutron in the $h_{11/2}$ subshell. This would give resulting angular momenta ranging from $\frac{11}{2} + \frac{3}{2} = 7$ to $\frac{11}{2} - \frac{3}{2} = 4$. Because the $s_{1/2}$ and $d_{3/2}$ neutrons have even parity and the $h_{11/2}$ neutron has odd parity, all of these couplings will give states with odd parity. If we examine the ^{130}Sn level scheme, we do indeed see several odd parity states with spins in the range of 4–7 with energies about 2 MeV. This energy is characteristic of what is needed to break a pair and excite a particle within a shell, and so we have a strong indication that we understand those states. Another possibility to form excited states would be to break one of the $h_{11/2}$ pairs and, keeping both members of the pair in the $h_{11/2}$ subshell, merely recouple them to a spin different from 0; according to the angular momentum coupling rules, the possibilities would be anything from $\frac{11}{2} + \frac{11}{2} = 11$ to $\frac{11}{2} - \frac{11}{2} = 0$. The two $h_{11/2}$ neutrons must be treated as identical particles and must therefore be described by a properly symmetrized wave function. This requirement restricts the resultant coupled spin to even values, and thus the possibilities are $0^+, 2^+, 4^+, 6^+, 8^+, 10^+$. There are several candidates for these states in the 2-MeV region, and here again the shell model seems to give us a reasonable description of the level structure.

A major exception to this successful interpretation is the 2^+ state at about 1.2 MeV. Restricting our discussion to the neutron states, what are the possible ways to couple two neutrons to get 2^+? As discussed above, the two $h_{11/2}$ neutrons can couple to 2^+. We can also excite a pair of $d_{3/2}$ neutrons to the $h_{11/2}$ subshell